The Pioneer's acceleration anomaly and Hubble's constant.

J.L. Rosales [1]

Abstract

The reported anomalous acceleration acting on the Pioneers spacecrafts could be seen as a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding space-time. We will show that, as a matter of fact, what has been detected in the experiments is just the cosmic expansion rate -the Hubble parameter H-, a feature of light signals. A relation for the reported annual term is obtained which depends on orbital parameters, leading to suggesting an analogy between the effect and Foucault's experiment, light rays playing a similar rôle in the expanding space than Foucault's Pendulum does while determining Earth's rotation.

Pioneer effect. A careful analysis of orbital data from Pioneer 10/11 spacecrafts has been reported[1],[2],[3] which indicates the existence of a very weak acceleration - approximately $a_p \simeq 8.5 \cdot 10^{-8} cm/s^2$ - apparently directed toward the Sun. The most conservative (or less adventurous) hypothesis is that these sorts of effects on the Pioneers's tracking data do not entail new physics and that the detected misfit must be due to some sophisticated (technological) reason having to do with the spacecraft configuration. However, the analysis in[1]-[3] seems to have ruled out many (perhaps all) of such technical reasons and the authors even claim having taken into account the accepted values of the errors in the planetary ephemeris, Earth's orientation, precession, and nutation.

Thus, in principle, a new effect seems to have unexpectedly entered the phenomenology of physics. On the other hand, if such an effect really exists (i.e., it can not be eliminated by data reanalysis) it would represent a

[1]Xerox Corp. The Document Company S.A.U. Ribera del Loira 16-18, E-28042 Madrid (Spain). E-mail: JoseLuis.Rosales@esp.xerox.com

violation of Birkhoff's theorem in general relativity for no constant acceleration at all is predicted by the Schwarzschild solution. One of the surprising features of the effect, as already pointed out in [1], is that it does not affect the planets (since no cumulative precession is observed in their trajectories) but only to spacecrafts (an apparently, strong violation of the equivalence principle.)

Position determinations procedures for planets and spacecrafts, however, differ. In Pioneer's experiment the procedure has to do with transmissions and receptions of light signals between an observer on Earth and the object whose coordinate position is to be determined, while for planets orbit perturbation theory is used to obtain cumulative result on the full trajectory. That is why Pioneer's coordinate determination allows not only to analyze the Solar System gravitational field in which the probes are certainly immersed, but also the nature of the non static metric background of the expanding space-time on which this gravitational field is acting on light itself.

We will show in this essay that the nature of the effect lays on the measurable properties of light rays that propagates within the expanding space background and that what has been detected in the experiments is just the cosmic expansion rate -the Hubble parameter H-, a feature of light signals. The results have, then, nothing to do with the existence of dark matter or any other source of local gravitational field.

The full effect is more conveniently understood when we notice that it deals properly on three different effects:

- The determination of an anomalous acceleration acting on the orbit, a_p, apparently directed toward the Sun (although the authors of [3] admit that the data are not provided of enough resolution as to determine whether the acceleration is either toward the Sun or toward the observer on Earth.) Moreover, a_p also has a small annual variation amplitude of the order of $\delta a_p \sim a_p \frac{1}{30}$.

- The anomalous Doppler effect which obtains a frequency drift acceleration, a_t. This is related with a_p upon assuming $a_t \equiv a_p/c$

- A new kind of clock acceleration, a_q, related but not directly correlated with the previous a_t, which can better be understood if there exists an expanding space-time. a_q corresponds to the detection (see [3]) of the

predicted effect in [4] that the observed time of travel for light signals in an expanding space has a shift of the kind

$$t \to t - \frac{1}{2}a_q t^2 \quad , \tag{1}$$

where $a_q = H$ is Hubble's constant.

We will now derive these three effects from the single assumption that there exist a expanding space background.

Let us start by considering a generic FRW metric

$$ds^2 = -c^2 dt^2 + \chi(t)^2 dr^2 \quad , \tag{2}$$

taking our units of space and time at the cosmological time t_1, we can write,

$$\chi(t) \simeq (\frac{t}{t_1})^{1-\delta} \quad , \tag{3}$$

where $\delta \ll 1$ is a constant depending on the density of the universe, and t_1 is the local "cosmic time".

Light geodesics satisfy

$$dl \equiv cdt = \chi dr \quad , \tag{4}$$

where dl is the length on the null cone.

On the other hand, we should be able to write our physical laws in such a way that the expansion of space time be scaled out. This requires using the radial function,

$$r_* \equiv \chi r \quad , \tag{5}$$

the metric then becoming

$$ds^2 = -(1 - \frac{r_*^2 H^2}{c^2})c^2 dt^2 + dr_*^2 - 2r_* H dr_* dt \quad . \tag{6}$$

where the local Hubble parameter is

$$H = \frac{d}{dt}\log(\chi) \quad . \tag{7}$$

Since these are not synchronous coordinates (for $g_{0r_*} \neq 0$), we define the radial vector

$$\vec{g} = -\frac{r_* H/c}{1 - r_*^2 H^2/c^2} \vec{r}_1 \ , \tag{8}$$

so that the space like element, as measured by some local observer, is the embedded three dimensional metric within the global space time in this manifold (see e.g. [5])

$$dl_*^2 \equiv \gamma_{r_* r_*} dr_*^2 = (g_{r_* r_*} - g_{00} g^2) dr_*^2 = \frac{dr_*^2}{1 - r_*^2 H^2/c^2} \ . \tag{9}$$

Me may now compare the length, l_* as measured in the locally scaled coordinates, with that on the light cone. In order to do this, notice that one might also have obtained, after (4) and (5), the following equation for the null cone,

$$dl = dr_*(l_*) - r_*(l_*) H \frac{dl}{c} \ , \tag{10}$$

whose solution - using (9), and noting that $\dot{H} \sim O(H^2)$- is

$$l = \frac{c}{H} \log\{1 + \sin(\frac{Hl_*}{c})\} \simeq l_* - \frac{Hc}{2} (\frac{l_*}{c})^2 + O(H^2) \ . \tag{11}$$

Which is Equation (1) and represents the measure of the space time curvature on the local null cone.

On the light cone, an effective quadratic in time term to the light time must be added, thus mimicking a line of sight acceleration of the spacecraft. The constant value of the effective residual acceleration directed toward the center of coordinates is

$$\kappa = Hc \ . \tag{12}$$

($l = l_* - \frac{1}{2}\kappa(l_*/c)^2$.) This is the acceleration observed in Pioneer 10/11 spacecrafts. From the value reported in [1] we get a value for the Hubble parameter

$$H = \frac{\kappa}{c} \simeq 83 km/s \cdot Mpc \ . \tag{13}$$

The result depends on the non synchronous character of the used coordinates in Equation (6). Thus, the measured difference between the values of

the universal time corresponding to two simultaneous events that take place in two different points of space may be computed as[6]

$$\delta t = \frac{1}{c}\int_A^B \vec{g}\cdot d\vec{r} \ . \tag{14}$$

Which is *path dependent*.

This gives, for the spacecrafts,

$$\delta t = \frac{1}{c}\int_0^{l_*} \vec{g}\cdot d\vec{r} \simeq -\frac{1}{2}H(l_*/c)^2 \ . \tag{15}$$

The total measurable time for a light signal reaching Observer's position is given approximately by

$$t_* \simeq l_*/c - \frac{1}{2}H(l_*/c)^2 \ , \tag{16}$$

which could also be interpreted in terms of a measurable length given by

$$l = c_* t_* \simeq l_* - \frac{Hc}{2}(\frac{l_*}{c})^2 + O(H^2) \ , \tag{17}$$

which is just our previous result in Eq. (11).

Yet, the interpretation of our result in terms of the very existence of an asynchronism in the universal time coordinate (depending on the path) between points on the expanding manifold, also allows for understanding that closed trajectories could not be used to detect the expansion of the space time; for, if we try to synchronize clocks using a closed path, we obtain

$$\delta t = \frac{1}{c}\oint \vec{g}\cdot d\vec{r} = 0 \ . \tag{18}$$

Which explains why *Pioneer effect* leads to no cumulative measurable effects on the orbits of the planets.

Now, let us obtain the relation between a_p and a_t. For a monochromatic light ray of wave number k and frequency ω in the metric (6), one easily obtains (see e.g. [6] [2]),

$$k_{r_*} = \frac{\omega}{c}\{\frac{\gamma_{r_* r_*}}{[-g_{00}]^{1/2}}\frac{dr_*}{dl_*} + g_{r_*}\} \ , \tag{19}$$

[2]notice that we use different sign convention in (6) than in ref. [6]

which reads as

$$\omega = ck(1 + \frac{Hr_*}{c}) + O(H^2) \quad . \tag{20}$$

Correspondingly,

$$\dot{k} = -\frac{\partial \omega}{\partial r_*} = -kH \ , \tag{21}$$

or

$$k = k_0(1 - Ht) + O(H^2) \ , \tag{22}$$

and,

$$\dot{r_*} = \frac{\partial \omega}{\partial k} = c(1 + \frac{Hr_*}{c}) \ ; \tag{23}$$

finally, from Equations (20) and (22) at the observer's position ($r_* = 0$), we obtain

$$\omega = \omega_0(1 - Ht) \ , \tag{24}$$

where $\omega_0 = k_0 c$. This corresponds to the measured a_t (see [3]), i.e.,

$$H = a_t = 2.8 \cdot 10^{-18} s^{-1} \ . \tag{25}$$

Moreover, from Equation (23), an observer might infer the presence of an effective Postnewtonian gravitational potential

$$\phi_H = Hcr_* \ , \tag{26}$$

where r_* is *the coordinate position of the emitter with respect to that observer*. This leads to an apparent vector acceleration that must be added to the orbital major forces

$$\vec{a} = -\frac{\partial}{\partial \vec{r_*}} \phi_H = -Hc\frac{\vec{r_*}}{r_*} \ . \tag{27}$$

Let us now determine $\vec{a}_p$, i.e., that part of the acceleration directed toward the Sun. [3] Since $\vec{r_*} = \vec{R}_* - \vec{R}_E$, where $\vec{R}_E$, and $\vec{R}_*$, are Earth's heliocentric coordinate vector and Pioneer's, one obtains for large R_*

$$\vec{a}_p \approx -Hc(1 + \zeta(t) \cos \gamma_0) \frac{\vec{R}_*}{R_*} \ , \tag{28}$$

[3]the remaining component has no effects since its averaged vector value is always zero in a year.

$\zeta(t) = (R_E/R_*)\cos(2\pi t/T)$. Here $t = nT$, represents conjunctions and γ_0, is the spacecraft ecliptic latitude as measured from the Sun.

The periodic term pretty explains the observation of an additional Doppler drift, correlated with Earth's orbital parameters [2],[3]

$$\delta a_p \approx -Hc\zeta(t)\cos\gamma_0 \ , \tag{29}$$

which has maxima on conjunctions as reported in [2] and [3]. The corresponding total drift in one year period is

$$\frac{\Delta\omega}{\omega} = 2HT\frac{R_E}{R_*}\cos\gamma_0 \ , \tag{30}$$

for Pioneer 10 we can take $< \frac{R_E}{R_*}\cos\gamma_0 >_{orbit} \sim 1/30$ and the annual Doppler amplitude becomes

$$< \frac{\Delta\omega}{\omega} >_{orbit} \sim 5\cdot 10^{-12} \quad , \tag{31}$$

the value of the observed annual Doppler amplitude anomaly coincides with our estimates in Equation (31)[3]. Moreover, Equation (29) also explains the fact that at early times of the experiment, the annual term in the anomalous acceleration is largest (see Fig.17 in [3].)

Notice the remarkable character of our result in Equations (27) - (30). The Pioneer's annual effect depends on the cosine of the ecliptic latitude γ_0, which demonstrates that the annual term is a pure geometrically driven effect. This statement could be seen as a benchmark for future experiments. It also suggest an heuristic analogy with Foucault's experiment; there, the spin frequency depends on Pendulum's colatitude λ in a way entirely similar to Equation (30)- recall $\omega_{Foucault} = \omega_{Earth}\cos\lambda$.

We will finally illustrate the statement that the Pioneer effect is a property of light signals, i.e., there exist no additional gravitational field apart from the standard Newtonian one. In order to show this, let us introduce the following transformation in the coordinates of Equation (2) for $\delta \simeq 0$. It relates the Lorentzian metric with the Milne ($\chi = Ht$) space time:

$$dt = \frac{1}{(c^2\tau^2 - \tilde{r}^2)^{1/2}}[c\tau d\tau - \frac{\tilde{r}}{c}d\tilde{r}] \ , \tag{32}$$

$$dr = \frac{c^2}{H(c^2\tau^2 - \tilde{r}^2)}[\tau d\tilde{r} - \tilde{r}d\tau] \ . \tag{33}$$

Now, $ds^2 = 0$ leads to

$$\frac{d\tilde{r}}{d\tau} \simeq c \ , \tag{34}$$

i.e., $\tilde{r} = c\tau + \tilde{r}_0$ - independently of the values of H. This means that the effect can be exactly removed in these coordinates. This satisfactory fact agrees with the consequences of Birkhoff's theorem.

Acknowledgements. Thanks to the staff of Xerox Corporation (Spain). I also wish to thank to Dr.José Luis Sánchez-Gómez from the Universidad Autónoma de Madrid for stimulating discussions.

References

[1] J.D. Anderson et al. *Phys. Rev. Lett.* **81**, 2858, (1998).

[2] S.G. Turyshev et al. gr-qc/9903024.

[3] J.D. Anderson et al. *Phys. Rev.***D65**,082004, (2002) and gr-qc/0104064

[4] J.L. Rosales and J.L. Sánchez-Gómez gr-qc/9810085.

[5] A. Lichnerowicz, *Th. Rel. de la Grav. et de l' Elec.,* Masson (1955).

[6] L.D. Landau, E. M. Lifshitz *Classical Field Theory.*